

Exercise Grading

Some exercises are based on proofs that are perhaps too hard. In those cases I give you a proof sketch and you fill in missing parts.

For each such exercise, grading is pass/fail based on whether a strict majority of parts is completed. For example, in an exercise with four parts, completing two or fewer is “fail” and completing three or more is “pass”.

Exercise: complete the sketch of a proof of completeness below. Complete the **BOXED** parts.

To prove: If a set of clauses Γ is unsat then there is a refutation for it

Proof Sketch (Davis + Putnam)

Remove trivial clauses of form $p \vee \neg p$.

* Assume Γ is finite and unsat.*

Γ must be non-empty. **why?**

Prove *current case* by induction on number of propositions occurring in Γ .

Base

Induction Step Suppose p occurs in Γ .

Define $\Gamma_p^+ := \{ C \in \Gamma : p \text{ occurs positively in } C \}$
 $\Gamma_p^- := \{ C \in \Gamma : p \text{ occurs negatively in } C \}$
 $\Gamma_p^0 := \{ C \in \Gamma : p \text{ doesn't occur in } C \}$

Define $\Gamma_p := \Gamma_p^0 \cup \{(C \cup D) \setminus \{p, \neg p\} : (C, D) \in \Gamma_p^+ \times \Gamma_p^-\}$

Observe: p doesn't occur in Γ_p .

Lemma: Γ_p is unsat.

Proof: For contradiction, suppose $v \models \Gamma_p$.
Let v' agree with v on all but p .

By relevance lemma, $v' \models \Gamma_p$.

W.L.O.G. assume $v(p) = 1$ and $v'(p) = 0$.

$$\begin{array}{ll} v \models \Gamma_p^0 & v' \models \Gamma_p^0 \\ v \models \Gamma_p^+ & v' \models \Gamma_p^- \end{array}$$

Finish proving this lemma

Finish the induction step by defining a refutation.

Complete the proof by arguing for the infinite case.

Hint: use Compactness.